

Tracial and Arens algebras associated with finite von Neumann algebras

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Let M be a von Neumann algebra with the positive cone M^+ and μ be a fixed faithful normal finite (f.n.f.) trace on a von Neumann algebra M . Following [1] consider the intersection

$$L^\omega(M, \mu) = \bigcap_{p \in [1, \infty)} L_p(M, \mu).$$

It is known [2] that $L^\omega(M, \mu)$ is a complete locally convex $*$ -algebra with respect to the topology generated by the system of norms $\{\|\cdot\|_p\}_{p \in [1, \infty)}$.

Denote by \mathcal{F} the set of all f.n.f. traces on M and from now on suppose that $\mathcal{F} \neq \emptyset$. Consider the space

$$M_f = \bigcap_{\mu \in \mathcal{F}} \bigcap_{p \in [1, \infty)} L_p(M, \mu) = \bigcap_{\mu \in \mathcal{F}} L^\omega(M, \mu).$$

On the space M_f one can consider the topology t , generated by the system of norms $\{\|\cdot\|_p^\mu : \mu \in \mathcal{F}, p \in [1, \infty)\}$.

Theorem 0.1. *(M_f, t) is a complete locally convex topological $*$ -algebra.*

The topological $*$ -algebra M_f is called the *finite tracial algebra* with respect to the von Neumann algebra M .

Theorem 0.2. *Let M be a finite von Neumann algebra and suppose that $\mathcal{F} \neq \emptyset$ is the family of all f.n.f. traces on M . The following conditions are equivalent:*

- (i) $M_f = L^\omega(M, \mu)$ for some (and hence for all) $\mu \in \mathcal{F}$;
- (ii) (M_f, t) is metrizable;
- (iii) (M_f, t) is reflexive;
- (iv) the center Z of M is finite-dimensional, i.e. $M = \sum_{i=1}^m M_i$, where all M_i are I_n -factors or II_1 -factors.

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Minimal projections with respect to numerical radius

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In this talk we survey some results on minimality of projections with respect to numerical radius. We note that in the cases L^p , $p = 1, 2, \infty$, there is no difference between the minimality of projections measured either with respect to operator norm or with respect to numerical radius. However, we give an example of a projection from l_3^p onto a two-dimensional subspace which is minimal with respect to

norm, but not with respect to numerical radius for $p \neq 1, 2, \infty$. Furthermore, utilizing a theorem of Rudin and motivated by Fourier projections, we give a criterion for minimal projections, measured in numerical radius.

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On Montel-Popoviciu Theorem and the closures of graphs of discontinuous polynomial functions

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In this address the regularity properties of solutions of Fréchet’s functional equation are shown under several different perspectives. On the one hand, we review the basic regularity results both for Cauchy’s and Fréchet’s functional equations so that the address will serve as an introduction to this nice topic and on the other hand, we state Montel’s theorem and prove, with our own arguments, the refined version of the result, due to Popoviciu. Using the tools we have just introduced for the proof of Montel-Popoviciu Theorem, we show that, if $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a discontinuous polynomial function, the topological closure of its graph $G(f)$ (as a subset of \mathbb{R}^{n+1}) contains an unbounded open set.

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Harmonic functions on hypergroups

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We introduce harmonic functions on hypergroups and discuss their properties, including the Liouville theorem and the Harnack inequality.

Extreme points of space semi-additive functionals

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The notion of normal functors on the category *Comp* of compact Hausdorff topological spaces and their continuous mappings it was introduced in 1997 by E. Shchepin. The following functors are well investigated among normal functors: the functor P of probability measures; the functor exp of a hyperspace. In 1998, T. Radul introduced the functor O of weakly additive order-preserving normed functionals in the category of compacts and he proved that the functor O is semi-normal. In 2008, D. Davletov and G. Djabbarov investigated the functor of semi-additive functionals. A general form of semiadditive functionals were given. Also had investigated categorical properties of the functor of semiadditive functionals OS .

It is well-known that the extreme boundary of the space of probability measures on compactum is the set of all Dirac measures on this compactum which is homeomorphic to initial compactum. This property plays crucial rule in the investigations of geometric properties of the functor of probability measures on compactum. The structure of the extreme boundary of the space semi-additive functionals on compactum is still not described.

In this talk we will discuss an extreme boundary of the convex compact set $OS(X)$. We will give some class semi-additive functionals which are extreme point in $OS(X)$.

On the distance to normal elements in C^* -algebras of real rank zero

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In 1993, Huaxin Lin showed that the distance from an $n \times n$ -matrix A , $\|A\| \leq 1$, to the closest normal $n \times n$ -matrix can be estimated from above by some function $F(\|[A, A^*]\|)$ such that $F(t) \rightarrow 0$ as $t \rightarrow 0$ uniformly in n . The original proof didn't give any other information on the behavior of F around zero. We obtain an order sharp estimate for the distance from a given bounded operator A on a Hilbert space to the set of normal operators in terms of $\|[A, A^*]\|$ and the distance to the set of invertible operators. For finite matrices, this implies that we can choose $F(t) = Ct^{1/2}$. A slightly modified estimate holds in a general C^* -algebra of real rank zero. The results are joint with Professor Yuri Safarov (King's College, London).

On universal enveloping locally C^* -algebras for a locally JB -algebra

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We show that for each locally JB -algebra

$$J \cong \varprojlim J_\alpha,$$

a projective limit of a projective family of JB -algebras J_α , $\alpha \in \Lambda$, there exists a unique (up to a local $*$ -isometry), universal enveloping locally C^* -algebra $LC_u^*(J)$, which is locally $*$ -isomorphic to a projective limit of the projective family of universal enveloping C^* -algebras $C_u^*(J_\alpha)$ of the family J_α , $\alpha \in \Lambda$, i.e.

$$LC_u^*(J) \cong \varprojlim C_u^*(J_\alpha).$$

(joint work with Oleg Friedman, University of South Africa, Pretoria, South Africa)

2-Local derivations on von Neumann algebras

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Given an algebra \mathcal{A} , a linear operator $D : \mathcal{A} \rightarrow \mathcal{A}$ is called a *derivation*, if $D(xy) = D(x)y + xD(y)$ for all $x, y \in \mathcal{A}$ (the Leibniz rule). Each element $a \in \mathcal{A}$ implements a derivation D_a on \mathcal{A} defined as $D_a(x) = [a, x] = ax - xa$, $x \in \mathcal{A}$. Such derivations D_a are said to be *inner derivations*. Recall that a map $\Delta : \mathcal{A} \rightarrow \mathcal{A}$ (not linear in general) is called a *2-local derivation* if for every $x, y \in \mathcal{A}$, there exists a derivation $D_{x,y} : \mathcal{A} \rightarrow \mathcal{A}$ such that $\Delta(x) = D_{x,y}(x)$ and $\Delta(y) = D_{x,y}(y)$.

The notion of 2-local derivations it was introduced in 1997 by P. Šemrl [5] and in this paper he described 2-local derivations on the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space H . A similar description for the finite-dimensional case appeared later in [4].

In [1] we suggested a new technique and have generalized the above mentioned results of [4] and [5] for arbitrary Hilbert spaces. Namely we considered 2-local derivations on the algebra $B(H)$ of all bounded linear operators on an arbitrary (no separability is assumed) Hilbert space H and proved that every 2-local derivation on $B(H)$ is a derivation. A similar result for 2-local derivations on finite von Neumann algebras was obtained in [3]. Finally, in [2] the authors extended all above results and give a short proof of this result for arbitrary semi-finite von Neumann algebras.

In this talk we present the following result.

Theorem. *Let M be an arbitrary von Neumann algebra. Then any 2-local derivation $\Delta : M \rightarrow M$ is a derivation.*

Note that our proof is essentially based on the analogue of Gleason theorem for signed measures on projection of von Neumann algebras.

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A functional representation of commutative symmetrical algebras, possessing an eigen-vector and acting on the Pontryagin P_1 space

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It is well known fact that in Pontryagin P_1 space with an indefinite metric all weakly closed algebras of bounded operators can be classified by 6 types (models): types 0, I , II_a , II_b , III_a , III_b , according to work of V. S. Shulman (Mat. Sbornik, 1972, 89, No 2). We are proving that only types 0, I , II_a and III_a could be represented as commutative symmetrical algebras. After that, we are stating and proving the theorem of functional representation for three types: 0, II_a and III_a . For type I the theorem had proven for the case of a single operator.

Factorizable completely positive maps and the Connes embedding problem

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The class of factorizable completely positive maps (originating in work of C. Anantharaman-Delaroche) has gained particular significance in quantum information theory in connection with the settling (in the negative) of the *asymptotic quantum Birkhoff conjecture*. More precisely, in joint work with Uffe Haagerup we proved earlier that every non-factorizable unital completely positive and trace-preserving map on $M_n(\mathbb{C})$, $n \geq 3$, provides a counterexample for the conjecture.

We will explain a recently established connection to the Connes embedding problem in terms of a newly formulated asymptotic property of factorizable maps.

Geometric Characterizations of Operator Algebras

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By a result of Blecher, Ruan, and Sinclair, it is known that unital operator algebras may be characterized among Banach algebras A by some simple matrix norm conditions. In this talk, I will explain how one can also characterize unital operator algebras by conditions on the “complete” holomorphic vector fields of the open unit ball of A . I will also discuss how further progress on the geometric theory

of operator algebras depends upon a better understanding of “partially” symmetric spaces.

Quadratic homeomorphisms of simplex S^2 and their trajectories

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Quadratic maps of simplex to itself occur in models of population genetics. In general these maps have the form:

$$V : x'_k = \sum_{i,j=1}^m P_{ij,k} x_i x_j, \quad k = 1, \dots, m, \quad (1)$$

where

$$S^{m-1} = \left\{ x = (x_1, \dots, x_m) \in \mathbf{R}^m : x_i \geq 0, \sum_{i=1}^m x_i = 1 \right\}$$

-($m - 1$)-dimensional simplex, $x = (x_1, \dots, x_m)$ and $P_{ij,k}$ -coefficient of inheritance, i.e.,

$$P_{ij,k} \geq 0, \quad P_{ij,k} = P_{ji,k}, \quad \sum_{k=1}^m P_{ij,k} = 1, \quad (i, j, k = 1, \dots, m).$$

Thus we have the operator $V : S^{m-1} \rightarrow S^{m-1}$. Volterra quadratic operator (2) is significant particular case of quadratic operators.

$$x'_k = x_k \left(1 + \sum_{i=1}^m a_{ki} x_i \right), \quad k = 1, \dots, m. \quad (2)$$

where $a_{ki} = -a_{ik}$, $-1 \leq a_{ki} \leq 1$.

It is known from [3], that Volterra maps are quadratic homeomorphism of simplex (1). We consider homeomorphism operator of all quadratic operator and dynamics of trajectory of there operators. In this talk we give description of case, when $m = 3$.

2-local *-homomorphisms on von Neumann algebras

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A not necessarily linear mapping T between Banach algebras A, B is said to be a *local homomorphism* if for every $a \in A$, there exists a homomorphism $\Phi_a : A \rightarrow B$ (depending on a), such that $T(a) = \Phi_a(a)$. One of the equivalent reformulations of the Gleason-Kahane-Żelazko theorem asserts that every unital linear local homomorphism from a unital complex Banach algebra A into \mathbb{C} is multiplicative. In 1980, Kowalski and Ślodkowski showed that at the cost of requiring the local behavior at two points, the condition of linearity can be dropped, that is, suppose A is a complex Banach algebra (not necessarily commutative nor unital), then every (not necessarily linear) mapping $T : A \rightarrow \mathbb{C}$ satisfying $T(0) = 0$ and $T(x - y) \in \sigma(x - y)$, for every $x, y \in A$, is multiplicative and linear.

Following the standard notation introduced by Šemrl in 1997, a (not necessarily linear nor continuous) mapping $T : A \rightarrow B$ is said to be a *2-local homomorphism* if for every $a, b \in A$ there exists a bounded (linear) homomorphism (respectively, a bounded isomorphism) $\Phi_{a,b} : A \rightarrow B$, depending on a and b , such that $\Phi_{a,b}(a) = T(a)$ and $\Phi_{a,b}(b) = T(b)$. *2-local Jordan automorphisms*, *2-local Jordan homomorphisms*, *2-local Jordan monomorphisms* and *2-local Jordan automorphisms* are defined in a similar fashion. The Kowalski-Słodkowski theorem establishes that every (not necessarily linear) 2-local homomorphism T from a (not necessarily commutative nor unital) complex Banach algebra A into the complex field \mathbb{C} is linear and multiplicative, and consequently, every (not necessarily linear) 2-local homomorphism T from A into a commutative C^* -algebra is linear and multiplicative.

In 1997, Šemrl proves that for every infinite-dimensional separable Hilbert space H , every 2-local automorphism $T : B(H) \rightarrow B(H)$ is an automorphism. In 2012, Ayupov and Kudaybergenov introduce new techniques to generalize Šemrl's theorem for arbitrary Hilbert spaces, showing that 2-local automorphisms on the algebra $B(H)$ on an arbitrary (no separability is assumed) Hilbert space H are automorphisms. Over twenty authors explored subtle variants and generalizations of the Kowalski-Słodkowski and Šemrl theorems in many different directions.

In this talk we present very recent and conclusive results on the study of 2-local $*$ -homomorphisms on C^* -algebras, showing that every (not necessarily linear) 2-local $*$ -homomorphism from a von Neumann algebra or from a compact C^* -algebra into another C^* -algebra is linear and a $*$ -homomorphism.

The Classification Problem of Leibniz algebras

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The talk concerns the classification problem of Leibniz algebras. The Leibniz algebra is a generalization of Lie algebra. Some of results of structural theory of Lie algebras can be easily extended to Leibniz algebras however the solution of others essentially depend on the fact that the Leibniz algebra is more general object than Lie algebra.

In the talk we intend to review the latest results on classification problem of Leibniz algebras, achievements have been made so far, approaches and methods implemented. The talk includes as well as a discussion on perspectives of the classification problem of the class of Leibniz algebras and its subclasses.

On Left-Symmetric Dialgebras

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The research deals with the structural theory of Left-Symmetric Dialgebras (LSD). This class of algebras has been introduced as a generalization of Left symmetric algebras (LSA). The structural theory of LSA is well developed, and there are many classification results in the literature. However, many structural problems of LSD are still remain untouched. In the paper we do first effort to study structure

theory of Left Symmetric Dialgebras: namely some notions have been introduced and two dimensional complex algebras are described. In order to motivate the study we present some interesting examples. Maple program "Check LSD" will be introduced and applied.

Keywords: Left Symmetric algebra; Left Symmetric dialgebra, isomorphism.

Elementary amenable groups have quasidiagonal C^* -algebra

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Rosenberg proved in 1987 that if the C^* -algebra of a discrete group is quasidiagonal, then the group is amenable, and he conjectured that the converse also holds. Using techniques from the classification of C^* -algebras and a description of elementary amenable groups due to Chou and Osin we confirm Rosenberg's conjecture for elementary amenable groups. This is a joint work with N. Ozawa and Y. Sato.

Triple derivations on von Neumann algebras

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It is well known that every derivation of a von Neumann algebra into itself is an inner derivation and that every derivation of a von Neumann algebra into its predual is inner. It is less well known that every triple derivation (to be defined) of a von Neumann algebra into itself is an inner triple derivation

We examine to what extent all triple derivations of a von Neumann algebra into its predual are inner. This rarely happens but it comes close. We prove a (triple) cohomological characterization of finite factors and a zero-one law for factors. Namely, we show that for any factor, the linear space of triple derivations into the predual, modulo the norm closure of the inner triple derivations, has dimension 0 or 1: It is zero if and only if the factor is finite. By studying the relation of triple derivations to commutators we show that every factor of type II_1 has outer derivations into its predual. (This is joint work with Robert Pluta.)

On classification of 5-dimensional solvable Leibniz algebras

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Leibniz algebras, a "noncommutative version" of Lie algebras, were introduced in 1993 by Jean-Louis Loday. During the last 20 years the theory of Leibniz algebras has been actively studied and many results on Lie algebras have been extended to Leibniz algebras. Particularly, in 2011 the analogue of Levi's theorem has been proven by D. Barnes. He showed that any finite-dimensional complex Leibniz algebra is decomposed into a semidirect sum of the solvable radical and a semisimple Lie algebra.

Note that semisimple Lie algebras is a direct sum of simple Lie algebras which are completely classified in fifties of the last century. Thus, the main issue in the

classification problem of finite-dimensional complex Leibniz algebras is to study the solvable part.

Since the description of the whole n -dimensional solvable Leibniz algebras seems to be unsolvable, we reduce our discussion to the restriction on their dimension.

It should be noted that several works were devoted to the description of low dimensional complex Leibniz algebras. In particular, the classifications of complex Leibniz algebras in dimensions less than five are obtained. Therefore, we reduce our attention to the classification of 5-dimensional Leibniz algebras. To describe of 5-dimensional solvable Leibniz algebras we use the method for describing solvable Lie algebras with given nilradical by means of non-nilpotent outer derivations of the nilradical.

Dominated Ergodic Theorem for isometries of non-commutative L_p -spaces, $1 < p < \infty, p \neq 2$

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We prove the dominated ergodic theorem for positive invertible isometries of the non-commutative L_p -spaces associated with finite von Neumann algebras. As a corollary, we obtain the individual ergodic theorem for such isometries.

Various types of Orbit Reflexivity

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An algebra of continuous linear maps is said to be *reflexive* if it has enough invariant subspaces to characterize it. Several notions of reflexivity have been studied during the last few decades. In this talk we will discuss some of them that are related to the notion of *orbit reflexivity*.

The orbit of a linear map T , $Orb(T)$, is defined as the semigroup generated by T and the identity linear map I . We define $OrbRef(T)$ to be the set of all continuous linear maps (operators) A such that, for every vector x , the vector $A(x)$ is in the closure of the set $Orb(T)x$. T is said to be orbit reflexive if $OrbRef(T)$ is the closure of $Orb(T)$ in the strong operator topology (SOT).
