

## **Szegö Limit Theorems With Varying Coefficients**

**Alain Bourget**

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Szegö's first limit theorem is a remarkable result concerning the asymptotic distribution of the eigenvalues of large Toeplitz matrices. In this talk, I will present several extensions of Szegö's results to large matrices with varying coefficients that satisfy a Cesaro-Nevai type condition. If time permits, I will also present some applications of our results to Jacobi matrices, and band-matrices in general.

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## **Optimal trace ideal properties of the restriction operator and applications**

**Rupert Frank**

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We discuss optimal trace ideal properties of surface restrictions of the Fourier transform. This is equivalent to generalizing the Stein-Tomas restriction theorem to systems of orthonormal functions with an optimal dependence on the number of functions.

As applications we deduce an optimal trace ideal version corresponding to the Strichartz bound for solutions of the time-dependent Schrödinger equation, a trace ideal version of the limiting absorption principle for Schrödinger operators and Lieb-Thirring bounds for eigenvalues of Schrödinger operators with complex potentials.

This talk is based on joint works with M. Lewin, E. Lieb and R. Seiringer and with J. Sabin.

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## **Measure of the spectrum of the extended Harper's model**

**Rui Han**

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In this talk, we will discuss the measure of the spectrum of the Extended Harper's model(EHM). The measure of the spectrum of the Harper's model, which in mathematics is better known as the almost Mathieu operator(AMO), is known to be  $|4 - 2a|$  where  $a$  is the coupling constant. The way of calculating the measure mainly relies on the analysis of the AMO with rational frequency and the continuity argument. Here we focus on how to calculate the measure of the spectrum of the EHM with rational frequency, therefore implying the result for the irrational frequency.

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## **Absolutely Continuous Branch of the Spectrum of Schroedinger Operator with Quasi-periodic Potential**

**Yulia Karpeshina**

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We consider  $H = -\Delta + V(x)$  in dimension two,  $V(x)$  being a quasi-periodic potential. We prove that the spectrum of  $H$  contains a semiaxis (Bethe-Sommerfeld

conjecture) and that there is a family of generalized eigenfunctions at every point of this semiaxis with the following properties. First, the eigenfunctions are close to plane waves  $e^{i(\vec{k}, \vec{x})}$  at the high energy region. Second, the isoenergetic curves in the space of momenta  $\vec{k}$  corresponding to these eigenfunctions have a form of slightly distorted circles with holes (Cantor type structure). It is shown that the spectrum corresponding to these eigenfunctions is absolutely continuous. A new method of multiscale analysis in the momentum space is developed to prove the results.

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### **Expansions for the energies of bound states of the one particle hamiltonians on lattices**

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The bound states of the hamiltonian  $h_\lambda$  of a particle moving on the  $d$ -dimensional lattice  $\mathbb{Z}^d$ ,  $d \geq 1$  and interacting with a external field  $\lambda v(x)$ ,  $x \in \mathbb{Z}^d$  are studied. Here  $\lambda$  is a coupling constant. The existence of the coupling constant threshold  $\lambda_{\min}$  is shown and an expansions for the energies of bound states at  $\lambda_{\min}$  are derived.

This is joint work with Saidakhmat N. Lakaev

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### **Unique continuation principle for spectral projections of Schrödinger operators and optimal Wegner estimates for random Schrödinger operators**

**Abel Klein**

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We prove a unique continuation principle for spectral projections of Schrödinger operators. Given a Schrödinger operator  $H = -\Delta + V$  on  $L^2(\mathbb{R}^d)$ , let  $H_\Lambda$  denote its restriction to a finite box  $\Lambda$  with either Dirichlet or periodic boundary condition. We prove unique continuation estimates of the type

$$\chi_I(H_\Lambda)W\chi_I(H_\Lambda) \geq \kappa \chi_I(H_\Lambda) \quad \text{with } \kappa > 0,$$

for appropriate potentials  $W \geq 0$  and intervals  $I$ . As an application, we obtain optimal Wegner estimates at all energies for one-particle and multi-particle continuous random Schrödinger operators with alloy-type random potentials.

#### **REFERENCES**

- [1] Abel Klein: Unique continuation principle for spectral projections of Schrödinger operators and optimal Wegner estimates for non-ergodic random Schrödinger operators. Comm. Math Phys. **323**, 1229-1246 (2013).
- [2] Abel Klein and Son T. Nguyen: Bootstrap multiscale analysis and localization for multi-particle continuous Anderson Hamiltonians. J. Spectr. Theory, to appear. arXiv:1311.4220

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### **The threshold effects for systems of the two and three-particles on lattices**

**Saidakhmat N. Lakaev**

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The Hamiltonians of systems of two resp. three quantum mechanical particles moving on the  $d$ -dimensional lattice  $\mathbb{Z}^d$ ,  $d \geq 1$ , and interacting via pairwise short-range potentials are considered.

The existence of eigenvalues for the two-particle Schrödinger operators  $h(k)$  depending on the quasi-momentum  $k \in \mathbb{T}^d = (-\pi, \pi]^d$  of two particles is proved.

The location of the essential and discrete spectra of the three-particle discrete Schrödinger operator  $H(K)$  depending on the three-particle quasi-momentum  $K \in \mathbb{T}^d$  is established.

If  $d = 3$ , the existence of infinitely many eigenvalues (Efimov's effect) of  $H(0)$  lying outside of the essential spectrum is proved and the corresponding asymptotics for the number of eigenvalues of  $H(0)$  is found. Moreover, for all nonzero values of the three-particle quasi-momentum  $K \in \mathbb{T}^3$  the finiteness of the number of eigenvalues of  $H(K)$  outside of the essential spectrum is proved and corresponding asymptotics for the number of eigenvalues of  $H(K)$  as  $K \rightarrow 0$  is obtained.

## The Spectral types of the Maryland Model

**Wencai Liu**

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In this talk, we give a precise description of the spectra of Maryland model  $(h_{\lambda, \alpha, \theta} u)_n = u_{n+1} + u_{n-1} + \lambda \tan \pi(\theta + n\alpha) u_n$  for all values of parameters, completing a program started about 30 years ago. We introduce an index  $\delta(\alpha, \theta)$  and show that  $\sigma_{sc}(h_{\lambda, \alpha, \theta}) = \{e : \gamma_\lambda(e) \leq \delta(\alpha, \theta)\}$  and  $\sigma_{pp}(h_{\lambda, \alpha, \theta}) = \{e : \gamma_\lambda(e) \geq \delta(\alpha, \theta)\}$ . Combining with the known fact  $\sigma_{ac}(h_{\lambda, \alpha, \theta}) = \emptyset$ , this gives the complete description of the three spectral types of Maryland Model. We also give a precise description of the set of eigenvalues, for all parameters. This is joint work with S. Jitomirskaya.

## Higher order Szegő theorems of arbitrary order

**Milivoje Lukic**

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We study the relation between a probability measure  $\mu$  on the unit circle and its sequence of Verblunsky coefficients  $\alpha$ . The CMV matrix determined by  $\alpha$  has  $\mu$  as its spectral measure, making this a spectral theoretic model closely related to Jacobi and Schrödinger operators.

Higher order Szegő theorems are equivalence statements relating decay conditions on  $\alpha$  with integral conditions on the absolutely continuous part of the measure. They have immediate corollaries establishing presence of absolutely continuous spectrum under appropriate decay conditions. We will present a higher order Szegő theorem of arbitrary order; this is the first known result of this form in the regime of very slow decay, i.e. with  $\ell^p$  conditions with arbitrarily large  $p$ .

## On the Frobenius-Perron theorem for a virtual level

**Konstantin Makarov**

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We discuss spectral properties of lattice Hamiltonians that generate positivity preserving semigroups on  $\ell^2(\mathbb{Z}^d)$ ,  $d \geq 3$ . We show that if a Hamiltonian has no discrete spectrum below the threshold, then the threshold is not an eigenvalue in dimensions  $d = 3, 4$ . We also obtain an extension of the Frobenius-Perron theorem for the threshold resonant states.

This is a joint work with S.N. Lakaev and Z.I. Muminov

## **Dominated splittings and the spectrum of almost periodic Jacobi operators**

**Christoph Marx**

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In recent years a dynamical system approach to the spectral theory of almost periodic Schrödinger operators proved to be extremely fruitful. One fundamental ingredient is a dynamical characterization of the spectrum due to R. A. Johnson, which determines the resolvent as the set of energies whose associated Schrödinger cocycles are uniformly hyperbolic. In this talk we present an extension of this theorem to almost periodic Jacobi operators. Whereas Schrödinger cocycles are always unimodular, Jacobi cocycles are in general non-invertible (“singular”), whence a different dynamical framework is called for. We will identify the notion “dominated splitting” as appropriate analogue of uniform hyperbolicity, suitable when passing from Schrödinger to (possibly singular) Jacobi operators.

## **Note on the spectrum of Schrödinger Operators on lattices**

**Zahriddin I. Muminov**

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The spectrum of the discrete Schrödinger operator  $-\Delta - V$  on the  $d$ -dimensional lattice  $\mathbb{Z}^d$  is studied. Here  $-\Delta$  stands for the standard discrete Laplacian and  $V$  is a multiplication operator by the function  $V(x) = \lambda\delta_{x0} + \mu \sum_{|s|=1} \delta_{xs}$ , where  $\lambda, \mu \in \mathbb{R}$  and  $\delta_{xs}$  is the Kronecker delta.

The dependence of the threshold resonance and eigenvalues on the parameters  $\lambda, \mu$  and  $d$  are explicitly derived.

## **Bounds on the density of states for Schrödinger operators with singular potentials**

**Chi Shing Sidney Tsang**

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We extend Bourgain and Klein’s results for bounds on the density of states measure for Schrödinger operators to the case of singular potentials, i.e., to potentials satisfying appropriate  $L^p$  conditions. (Joint work with A. Klein)

## **Spectral packing dimension for 1-dimensional quasiperiodic Schrödinger operators**

**Shiwen Zhang**

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In this talk, we are going to discuss the packing dimension of the spectral measure of 1-dimensional quasiperiodic Schrodinger operators. We prove that if the base frequency is Liouville, the packing dimension of the spectral measure will be one. As a direct consequence, we showed that for the critical and supercritical Almost Mathieu Operator, the spectral measure has different Hausdorff and packing dimension.

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