On generalized localization of Fourier inversion for distributions

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In this paper we study the behavior of spherical Fourier integrals and pointwise convergence and summability of Fourier inversion. We consider generalized localization principle which in classical L_p spaces was investigated by P.Sjölin, A.Carbery, F.Soria, and Sh.Alimov. Proceeding these studies in this paper we establish sharp conditions for generalized localization in the class of finitely supported distributions.

Spectral properties of a differential operator of Heun type arising in fluid dynamics

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We study the non-selfadjoint linear differential operator

$$Ly = \frac{d}{dt}\left((1 - a\cos t)y + b\sin t\frac{dy}{dt}\right)$$

acting in the Hilbert space $L^2(-\pi,\pi)$ that originated from a steady state stability problem in fluid dynamics. The operator L is of Heun type and involves two parameters a, b related to the hydrostatic pressure and capillary properties of the fluid. The results concern: (1) the properties of functions in the domain of definition of L, (2) conditions on a, b for the linear span of the Fourier basis e^{int} to be core of L, and (3) the matrix representation of the reduced resolvent of L in the Fourier basis. In particular, we prove that the reduced resolvent is compact and of trace class S_1.

Joint work with H. Volkmer

Spectral methods for analyzing large data

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There has been increasing demand to understand the data around us. The flood of social media requires new mathematics, methodologies and procedures to extract knowledge from massive datasets. Spectral methods are numerical linear algebra graph based techniques that uses eigenfunctions of a graph to extract the underlying global structure of a dataset. The construction of these, application dependent, graphs require new mathematical ideas that extend data representation, distance, topic modeling and sparsity. The product is often massive matrices that push the limits of matrix computation. This talk looks at applications to analyzing street gang networks, Twitter microblogs and content based search.

Optimal Eigenvalues of Laplace and Laplace-Beltrami Operators

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Since Lord Rayleigh conjectured that the disk should minimize the first Laplace-Dirichlet eigenvalue among all shapes of equal area more than a century ago, eigenvalue optimization problems have been active research topics with applications in various areas including mechanical vibration, electromagnetic cavities, photonic crystals, and population dynamics. In this talk, I will review some interesting classical problems and discuss our recent computational results for optimal eigenvalues of Laplace and Laplace-Beltrami Operators.

Complex powers of the Schrödinger operator with singular potential

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We consider in R^n $(n > 3)$ the Schrödinger operator

$$H(x,D) \;=\; -\Delta \;+\; V(x).$$

and suppose that potential V is smooth enough outside $S \subset R^n$, where

$$S \;=\; \bigcup_{k=1}^{l} S_k,$$

and S_k are smooth surfaces in R^n of dimension $n - m_k, \quad 3 \leq m_k < n$.
Let

$$\rho(x) \;=\; \operatorname{dist}(x,S) \;=\; \inf_{y\in S} |x-y|.$$

We suppose that for some $\tau,\ 0 \leq \tau < 1$,

$$|D^\alpha V(x)| \;\leq\; \operatorname{const}[\rho(x)]^{-1-|\alpha|}\{1 + [\rho(x)]^{-\tau}\},$$

for all multiindices α with $|\alpha| \leq n$.

As an example for such kind of operators we can take potential of many particle systems.
It is proved that imaginary powers of these operators are bounded in $L_p(R^n)$.
This fact and E.Steyn interpolation theorem gives us possibility to estimate of real fractional powers of the Schrödinger operator.

Diffuse Interface Models for Graph-Based Multiclass Data Classification

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We present generalizations of a binary diffuse interface model for classification of high-dimensional data. The original binary diffuse interface model adapts the Ginzburg-Landau continuum energy functional to a semi-supervised setup on graphs. The graph structure is used to encode a measure of similarity between data points. Motivated by total variation techniques, the method involves minimizing the functional on the graph. We develop two multiclass generalizations, one based on a scalar representation and other based on a vector-field representation. We demonstrate the performance of the two multiclass formulations on synthetic data as well as real benchmark sets, and discuss the consequences of using operators other than the graph Laplacian. The talk is based on joint work with Cristina Garcia-Cardona, Arjuna Flenner, Ekaterina Merkurjev and Andrea Bertozzi.

Feeling the shape of Brownian Motion on a manifold

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On any complete Riemann manifold we can define the Brownian motion as the diffusion process generate by the Laplace Beltrami operator. A more intuitive approach is furnished by the isotropic transport process, whose weak limit is BM when the mean free path tends to zero. Inverse questions can be formulated in terms of the mean exit time and its higher moments. In particular the five dimension process is BM, but there are counter-examples in dimensions six or higher.

The maximum principle of Alexandrov for very weak solutions

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We are concerned with maximum principle for second order elliptic operators in the form $Lu = Tr[\mathcal{A}(x)D^2u]$ where $A(x)$ is a $n \times n$ elliptic matrix.
We relax the assumption $u \in W^{2,n}_{loc}(\Omega)$ and we obtain several results for such solution which we refer to very weak solutions.
One of such classes is the Orlicz-Sobolev space $W^{2,\Theta}(\Omega)$ of functions u whose second derivatives satisfy

$$\int_\Omega \frac{|D^2u|^n}{\log^{\frac{1}{n}}(e + |D^2u|)} \;<\; \infty.$$

We prove that if $u \in W^{2,\Theta}(\Omega) \cap C(\overline{\Omega})$ is concave then

$$\sup_\Omega u \;\leqslant\; \sup_{\partial\Omega} u^+ \;+\; C \;\left\| \frac{Lu}{\det \mathcal{A}(x)^{\frac{1}{n}}} \right\|_{L^n(\Omega)}$$

where C is a positive constant depending only on n and diameter of Ω.
Similar results are established for uniformly elliptic operators L and continuous functions u that belong to the Grand Sobolev space $W^{2,n)}(\Omega)$.

This is a joint work with Gabriella Zecca

On the uniform convergence of Fourier series in a closed domain

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The uniformly and absolutely convergence in a closed domain of Fourier series associated with the Laplace operator first time studied by V.A. Il'in [3]. In [3] it was proved that if the repeated Laplacians of a function from $W_p^{\frac{N+2}{2}}$ satisfies boundary conditions, then its Fourier series converges uniformly in a closed domain $\overline{\Omega}$.

In [4] G.I. Eskin proved that Fourier series of a function from $W_p^{\frac{2N-1}{4}-\varepsilon}$ associated with the elliptic differential operator of the order $2m$ with the Lopatinsky boundary conditions converges in a closed domain. E.I. Moiseev [5] studied the problem for the elliptic operators of second order and obtained sharp eigenfunction estimations in a closed domain.

In [2] for the elliptic differential operator of order $2m$ with Lopatinsky boundary conditions it was proved that a condition $s > \frac{N}{2}$ is sufficient for the uniform convergence of the Riesz means of the Fourier series of the finite continuous function in a closed domain. Obtained condition for the Riesz means can be weaken ($s > \frac{N-1}{2}$) in case of Laplace operator. The convergence of the Riesz means below critical indexes, $s < \frac{N-1}{2}$, requires more smoothness from the function [1].

In the present paper we study the problem in the Nikolsky spaces with the mixed norm. The convergence of Fourier series (in compact subsets) of the functions from the spaces with the mixed norm studied earlier by Sozanov V.G. [6].

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Resonance and nonresonace with respect to the spectrum of elliptic operators

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Certain types of linear elliptic differential operators, $\mathcal{L}$, acting on functions defined on a domain, Ω, of Euclidean space with smooth boundary, $\partial\Omega$, have a discrete spectrum with respect to special types of boundary conditions; in other words, the eigenvalues of the problem

$$\mathcal{L}u = \lambda u \quad \text{in } \Omega, \quad \mathcal{B}u = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where $\mathcal{B}$ is linear boundary operator, form a discrete set. This is the case, for instance, in which $(\mathcal{L}, \mathcal{B})$ has a realization in a Banach space of functions, X, given by a closed linear operator L defined on a subspace $\mathcal{D}(L)$ of X, and such that $L\colon \mathcal{D}(L) \to X$ has a compact inverse.

For this kind of operators, the Fredholm alternative for the nonhomogeneous problem

$$\mathcal{L}u = \lambda u + f \quad \text{in } \Omega, \qquad \mathcal{B}u = 0 \quad \text{on } \partial\Omega \tag{2}$$

also holds true. This means that if λ in (2) is not in the spectrum of the problem (1), then the problem (2) is solvable for any $f \in X$. On the other hand, if λ is an eigenvalue of (1), the (2) is solvable if and only if f satisfies certain orthogonality conditions with respect to the eigenspace of λ.

In this presentation we explore extensions of the Fredholm alternative for the semi–linear problem

$$\mathcal{L}u(x) = \lambda u(x) + g(x, u(x)) \quad \text{in } \Omega, \qquad \mathcal{B}u = 0 \quad \text{on } \partial\Omega, \tag{3}$$

where g is a differentiable function defined on $\Omega \times \mathbb{R}$. In particular, we are interested in sufficient conditions for the solvability of the semi–linear problem in (3) for the case in which λ is an eigenvalue of (1) (this is called a resonance problem). We will also present recent existence and multiplicity results for the case in which λ is not an eigenvalue (nonresonance).

This is joint work with Leandro Recova of the Claremont Graduate University.

Using Spectral Clustering on Networks with Epidemic Diffusion

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Spectral clustering is widely used to partition graphs into distinct modules or communities. Existing methods for spectral clustering use the eigenvalues and eigenvectors of the graph Laplacian, an operator that is closely associated with random walks on graphs. We propose a new spectral partitioning method that exploits the properties of epidemic diffusion. An epidemic is a dynamic process that, unlike the random walk, simultaneously transitions to all the neighbors of a given node. We show that the replicator, an operator describing epidemic diffusion, is equivalent to the symmetric normalized Laplacian of a reweighted graph with edges reweighted by the eigenvector centralities of their incident nodes. Thus, more weight is given to edges connecting more central nodes. We describe a method that partitions the nodes based on the componentwise ratio of the replicator's second eigenvector to the first, and compare its performance to traditional spectral clustering techniques on synthetic graphs with known community structure. We demonstrate that the replicator gives preference to dense, clique-like structures, enabling it to more effectively discover communities that may be obscured by dense intercommunity linking.
