CORRIGENDUM TO: "A RANK INEQUALITY FOR THE TATE CONJECTURE OVER GLOBAL FUNCTION FIELDS"

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In [Lyo], we claimed the following:

Theorem 2.1. *For a smooth, projective, geometrically connected variety X over a global function field k, we have*

$$r^{(m)}_{\ell,k} = r^{(m)}_{\mathrm{an},k}$$

and thus

$$r^{(m)}_{\mathrm{alg},k} \leq r^{(m)}_{\mathrm{an},k}$$

for any $0 \leq m \leq \dim X$.

The equality $r^{(m)}_{\ell,k} = r^{(m)}_{\mathrm{an},k}$ is presently unknown in general and should be corrected to read

$$r^{(m)}_{\ell,k} \leq r^{(m)}_{\mathrm{an},k}.$$

In any case, the inequality

$$r^{(m)}_{\mathrm{alg},k} \leq r^{(m)}_{\mathrm{an},k},$$

which is the main focus of the paper, still holds since $r^{(m)}_{\mathrm{alg},k} \leq r^{(m)}_{\ell,k}$.

To see that one has $r^{(m)}_{\ell,k} \leq r^{(m)}_{\mathrm{an},k}$, one should correct the proof of Theorem 2.1 as follows. On p.104 of [Lyo], the line beginning "By (5b)..." is followed by a string of equalities. One should change the second equality to an inequality, so that string now reads:

$$\begin{aligned} -\mathrm{ord}_{s=1} L^S(\rho_\ell(m), s) &= -\sum_i \mathrm{ord}_{s=1} L^S(\rho_i, s) \\ &\geq \dim_{\bar{\mathbb{Q}}_\ell} (V_\ell(m) \otimes \bar{\mathbb{Q}}_\ell)^{\Gamma_k} \\ &= \dim_{\mathbb{Q}_\ell} V_\ell(m)^{\Gamma_k} \\ &= r^{(m)}_{\ell,k}. \end{aligned}$$

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The relevant point in introducing the inequality is that the order of the pole of $L^S(\rho_\ell(m), s)$ at $s = 1$ (which is $r^{(m)}_{\mathrm{an},k}$ by definition) is measuring the dimension of the Γ_k-invariants of the *semisimplification*, which in general is only an upper bound for the dimension of the Γ_k-invariants of the original representation.

Remark 1. If one knows that the Galois representation ρ_ℓ is semisimple (an assertion that is sometimes called the *Serre–Grothendieck Conjecture*), then one can deduce the stronger conclusion that $r^{(m)}_{\ell,k} = r^{(m)}_{\mathrm{an},k}$. In particular, this is known for products of curves and abelian varieties (by Zarhin [Zar] when k is a global function field and by Faltings [Fal] when k is a number field). Thus Proposition 6.1 is correct as stated, but one should note the use of semisimplicity of ρ_ℓ in its proof.

Remark 2. The corollary for the integers $r^{(m)}_{\mathrm{an},L}$ described in §2 does utilize the supposed equality $r^{(m)}_{\ell,k} = r^{(m)}_{\mathrm{an},k}$. Without this equality, one can only conclude that $r^{(m)}_{\mathrm{an},L} \geq r^{(m)}_{\mathrm{alg},L} \geq 1$ for all finite L/k, so the last three assertions in that section are still conjectural in general.

References

[Fal] G. Faltings. Endlichkeitssätze für abelsche Varietäten über Zahlkörpern. *Invent. Math.* **73** (1983), 349–366.

[Lyo] C. Lyons. A rank inequality for the Tate conjecture over global function fields. *Expo. Math.* **27** (2009), 93–108.

[Zar] J. G. Zarhin. Endomorphisms of Abelian varieties over fields of finite characteristic. *Izv. Akad. Nauk SSSR Ser. Mat.* **39** (1975), 272–277, 471.

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