Screw Theory and Kinematics of multi-link Manipulators

Mariia Grushina
Faculty Advisor: Prof. T. Murphy
California State University Fullerton

Introduction

Robotics is an integral part of human life, with robot manipulators being the most popular type. It is well known that for two rigid body positions P_1 and P_2, there are infinitely many possible trajectories between them [6]. Screw theory simplifies this: any two positions can be achieved via a single screw motion, making it ideal for manipulator kinematics. Our project aims to explicitly compute this motion for a specific Robotic manipulator: the KUKA KR 6 R900-2.

Figure 1. The KUKA robotic manipulator

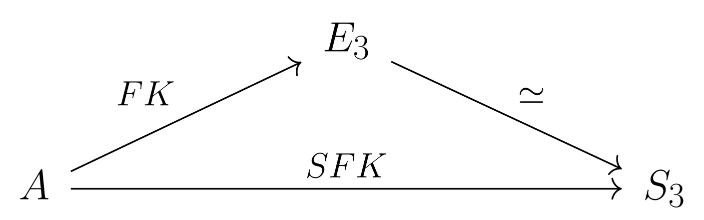
Key Features of Screw Theory

- *Universality & Uniqueness*: Any rigid body motion is realized by a unique screw motion.
- *Standard Formula*: Combines rotation and translation via a general group-theoretic description.
- *Group Structure*: all motions form the group $\mathcal{V}_3$, isomorphic to $\mathcal{E}_3^0$ (Euclidean group).
- *Mathematical Abstraction*: Enables differential-geometric formulations [7].

Work Process

The work process involves deriving the forward kinematics equations for the 6-link manipulator. The key steps are:

- Reformulation of the theory of rigid body motion in the language of the group of screw motion.
- Standard formulation of the forward kinematics problem (FK).
- Reformulated in the language of screw motions: realizing the movement of the manipulator from the initial position to the final one, as a function of manipulator's angles.



Step 1. Screw Parameters

Screw motion is a combination of rotation around an axis and translation along the same axis. It is described by:

$$R_{\vec{n}}^{\phi}(\vec{r}) + \vec{a} = R_{(\vec{n_1};\vec{c})}^{\psi}(\vec{r}) + k\vec{n_1}, \tag{1}$$

- $\vec{n_1}$ is the unit direction vector of the rotation axis
- $\vec{c}$ is the radius vector of a point belonging to the rotation axis with the direction vector $\vec{n_1}$, such that $\vec{c} \perp \vec{n_1}$.
- ψ is the rotation angle, k is the coefficient of the unit vector $\vec{n_1}$, which determines the length of the translation vector.

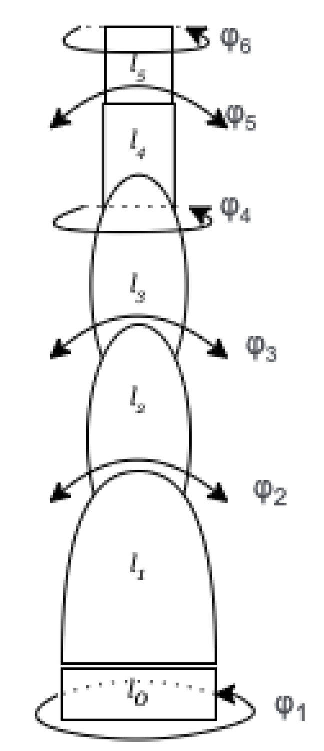
Main Theorem: Screw Parameters

$$\psi = \phi;\ k = \vec{n} \cdot \vec{a};\ \vec{c} = \frac{1}{2}(\vec{a} - (\vec{n} \cdot \vec{a})\vec{a} - (\vec{a} \times \vec{n}) \cot \frac{\psi}{2}). \tag{2}$$

Step 2. Forward Kinematics

Let us call the position of the manipulator, in which all its links are straightened and oriented along the vertical axis OZ directed upward, initial. Let's introduce the following angles: φ_1, φ_2, ...,φ_6, each of which is responsible for the rotation of the corresponding link of the six-link manipulator.

Let the point O', located at the end of the sixth link of the manipulator, be responsible for the position of the gripper, and the vector $\vec{r}$ emanating from the point O', for its orientation. We will observe changes in the position and orientation of the manipulator's grip during a sequential change in the values of the angles φ_i. As a result, we find the resulting rotation matrix and translation vector, under the influence of which the vector $\vec{r}$ will coincide with the vector $\vec{r_6}$, which is responsible for the final orientation of the gripper, and the point O' will coincide with the point O_6 , responsible for the final position of the end of the manipulator.



Main Theorem: Screw Parameters

Denoting the resulting rotation matrix by R_r and the resulting vector of translation of the gripper by $\vec{O_r}$, then

$$R_r = R_{n6}^{\varphi 6} \cdot R_{n5}^{\varphi 5} \cdot R_{n4}^{\varphi 4} \cdot R_{n2}^{\varphi 3} \cdot R_{n2}^{\varphi 2} \cdot R_{\vec{e_z}}^{\varphi 1}, \tag{3}$$

$$\vec{O_r} = \vec{O_6} - \vec{O_1} \tag{4}$$

Step 3

To derive the screw parameters as functions of the manipulator joint angles, we proceed as follows: First, we compute the resultant rotation angle φ, the unit direction vector of the rotation axis $\vec{n}$, and the translation vector $\vec{a}$ using Equations (3) and (4). Substituting these into Equation (2) yields the screw parameters expressed in terms of the joint angles φ_i.

Example 1: Screw example

To move the manipulator's grip from its initial z-aligned position to the target pose:

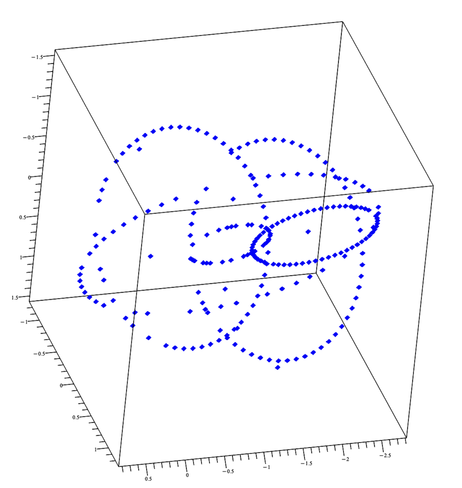
$$\vec{O_g} = \begin{pmatrix} -0.016 \\ -0.289 \\ 1.288 \end{pmatrix}, r_g = \begin{pmatrix} -0.18 \\ -0.324 \\ 0.928 \end{pmatrix}$$

the following parameters of the screw will be needed:

$$\psi = 47.31, k = 0.0335, c = \begin{pmatrix} 0.315 \\ -0.13 \\ -0.143 \end{pmatrix}$$

Results

While screw motion is universal, there are some points in the space where a unique screw motion may not exist. In other words, there are two different choice of coordinates realizing the same point in space. Those points are presented on the following diagram. Note these singularities fall into circles: this comes from the fact the screw motion is described using spherical coordinates. For all other points in the phase space **the screw exists** and it is **unique.**



However, if we put the constraints on the angles that are mentioned in the KUKA manual

$$\varphi_1 = \pm 170;\quad \varphi_2 = -190/45;\quad \varphi_3 = -120/156;\quad \varphi_4 = \pm 185;\quad \varphi_5 = \pm 120;\quad \varphi_6 = \pm 350;$$

the screw is unique. Therefore any movement in the manipulator's working space can be accomplished with a single screw motion.

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